Frascati March 28 , 1994

Dear Mr.Jerry Horn
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The 25th of March I attended a Burr-Brown's Applications Seminar, held in Rome, Italy.
We discussed, among other topics, the undersampling technique. About that, I communicated to the relators two formulae that would be useful to add to the seminar book and elsewhere for practical use. Mr. Jason Albanus suggested to me your name to ask about it.
If we know the lowest f_L and the highest f_H frequency of the input spectrum, we can put the basic principle (e.g. at p.232 of the seminar book) in the form $n\frac{f_S}{2} < f_L$ and $f_H < (n+1)\frac{f_S}{2}$ with $n = 0, 1, 2, \ldots$.
To respect to f_S we have $f_S < \frac{2f_L}{n}$ and $f_S > \frac{2f_H}{n+1}$, hence

$$\frac{2}{n+1} f_H < f_S < \frac{2}{n} f_L \qquad (1^{st} \text{ useful formula})$$

which is the interval of the allowed sampling frequencies.
The above inequalities imply $\frac{f_L}{n} > \frac{f_H}{n+1}$, i.e. $\frac{1}{n} > \frac{f_H}{f_L} - 1$ and finally

$$n < \frac{f_L}{f_H - f_L} \qquad (2^{nd} \text{ useful formula})$$

that gives us a criterion to choose n(integer). Besides we can rewrite the formula at p.231 as $f_{SAMPLED} = \left| f_{INPUT} - \left\lfloor \frac{n+1}{2} \right\rfloor f_S \right|$ where the symbol $\left\lfloor \frac{n+1}{2} \right\rfloor$ stands for "floor": the greatest integer not greater than $\frac{n+1}{2}$.
Furthermore, the order of the spectral bins of the input spectrum is preserved or inverted in the sampled spectrum, depending on the parity of n: the order is preserved if n is even, inverted if n is odd.
For example let $f_L = 24.6MHz$ and $f_H = 27MHz$,$BW = 2.4MHz$. We have
$n < \frac{24.6}{2.4} = 10.25$
Choosing $n = 10$ we obtain

$4.\overline{90}MHz < f_S < 4.92MHz$

We could choose $f_S = 4.91MHz$.
The sampled spectrum would be in the band
$f_{L_{SAMPLED}} = |24.6 - 5*4.91| = 0.05MHz$

$f_{H_{SAMPLED}} = |27 - 5*4.91| = 2.45MHz$

Because n is even, the spectral order of the input spectrum is preserved in the sampled spectrum.
Of course we could also use in principle $n = 0, 1, 2, \ldots,$ until 9. For $n = 0$ we would obtain $54MHz < f_S$, the Nyquist rate; for $n = 1$ $27MHz < f_S < 49.2MHz$ and so on up to 9: no other sampling frequencies intervals are allowed if we wants to avoid the folding of the sampled spectrum: e.g. $f_S = 4.89MHz > 2BW$ is not allowed.
I would appreciate your comments.

Yours sincerely,

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