Frascati December 7 , 1991.

Dear EDN Signals & Noise Editor,

The reference is my letter of October 10, 1991.
I want to add some information to clarify the expressions

$$\frac{2}{n+1} f_{INHIGH} < f_{SAMPLING} < \frac{2}{n} f_{INLOW} \quad (1)$$

$$n_{INTEGER} \leq \left\lfloor \frac{f_{INLOW}}{f_{INHIGH} - f_{INLOW}} \right\rfloor \quad (2)$$

in which the symbol $\lfloor --- \rfloor$ means "the whole part of ---" and $n_{INTEGER} = n$ of the expression (1).
This expression for $n_{INTEGER}$, when the band of the input signal is a submultiple of f_{INLOW}, as in the example of the article by Kirsten & Tarlton, EDN June 21,1990, implies that

$$\max(n_{INTEGER}) = \frac{f_{INLOW}}{f_{INHIGH} - f_{INLOW}}$$

This choice produces an alias which is unimportant in the example chosen by Kirsten & Tarlton, as they explicitly stated, but unacceptable in situations in which we are interested in precise measurements even of the spectral borders of a generic input band.
If we want to avoid that kind of alias we have to choose $n_{INTEGER}$ as

$$n_{INTEGER} < \frac{f_{INLOW}}{f_{INHIGH} - f_{INLOW}} \quad (3)$$

For the sake of precision the expression (1) implies (3) while (2) is implied by (1) with ≤ instead of <.
It is straightforward to prove the expressions (1) and (3) starting from the basic concept that our input band must be between $n\frac{f_{SAMPLING}}{2}$ and $(n+1)\frac{f_{SAMPLING}}{2}$.
We can immediately put the above statement in the form

$$\begin{cases} n\frac{f_{SAMPLING}}{2} < f_{INLOW} \\ f_{INHIGH} < (n+1)\frac{f_{SAMPLING}}{2} \end{cases} \quad (4)$$

hence it is a fortiori

$$f_{INHIGH} < f_{INLOW} + \frac{f_{INLOW}}{n}$$

and eventually we obtain the expression (3), while we have (1) directly from (4).

Bibliography:

a) Reference data for radio engineers, 5th Edition, 1970, Howard W. Sams & Co.-ITT :at p.21-14 there is a version of (1) in the time domain.

b) Bendat J.S. & Piersol A.G., Random data: Analysis and

measurement procedures, 1971, Wiley-Interscience : at p. 230 there is an illustration of the aliasing and an expression equivalent to $f_{ALIASED}=|f_{SIGNAL}-nf_{SAMPLING}|$.

c) Brigham E. O., The fast Fourier Transform, 1974, Prentice-Hall, New Jersey : At p.87 the problem 5-4 contains a formula for the critical sampling frequency obtainable from (4) with the substitutions $n=n'-1$ and $\leq$ instead of $<$.

Yours sincerely,

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