Frascati October 10, 1991

Dear Signals & Noise Editor,

I am referring to the article entitled "Undersampling reduces data acquisition costs for select applications", by Jeff Kirsten and Tarlton Fleming, published on EDN June 21, 1990 and again in July 1991 on Electronic Engineering.
I have been using the undersampling technique since long time ago (A.Ricotta, Some remarks on the sampling and processing of SODAR data, Technical Report, IFA-CNR, July 1983), and I have found useful to use two explicit formulae to quickly compute **all** the permitted intervals of sampling frequencies when f_{INLOW} and f_{INHIGH} of the bandpass input signal are done.
We can write

$$\frac{2}{n+1} f_{INHIGH} < f_{SAMPLING} < \frac{2}{n} f_{INLOW}$$

where

$$n_{INTEGER} \leq \left[\frac{f_{INLOW}}{f_{INHIGH} - f_{INLOW}} \right]$$

so that $n_{INTEGER} = 0, 1, 2, \ldots, \max(n_{INTEGER})$.
At $\max(n_{INTEGER})$ corresponds the interval of the minimal sampling frequencies.
Furthermore, if we choose $n_{INTEGER}$ even, the aliased input signal preserves its spectral order, while for $n_{INTEGER}$ odd, the order is reversed.
For example let $f_{INLOW} = 500kHz$ and $f_{INHIGH} = 550kHz$ as in the cited article.
From the above inequalities we obtain

$$n_{INTEGER} \leq \frac{500}{50} = 10 \quad \text{(exactly)}$$

hence $n_{INTEGER} = 0, 1, 2, 3, \ldots, 10$: in some applications we must reject the exact maximum $n_{INTEGER} = 10$ to avoid to alias the spectral borders of the input signal with 0 or $\frac{f_{SAMPLING}}{2}$.

From each value $n_{INTEGER}$ we can compute an interval of permitted sampling frequencies:
$n_{INTEGER} = 0 \Rightarrow 1100kHz < f_{SAMPLING}$ (Nyquist)

$n_{INTEGER} = 1 \Rightarrow 550kHz < f_{SAMPLING} < 1000kHz$
and eventually
$\max(n_{INTEGER}) = 10 \Rightarrow 100kHz < f_{SAMPLING} < 100kHz$
which is the interval of the minimal permitted frequencies (only one in this case). Because 10 is even the spectral order is preserved.
Generally we choose $\max(n_{INTEGER})$ and then the minimum allowed

$f_{SAMPLING}$, but for particular applications is useful to have the possibility to select different values of $f_{SAMPLING}$ and also of $n_{INTEGER}$.

Yours sincerely,

Address:
Angelo Ricotta
IFA-CNR
POB 27
00044 Frascati
ITALY